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A Technique for Dynamic Modelling of a Two-link Flexible Manipulator

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Abstract— **The paper presents dynamic modelling of a two-link flexible manipulator model. An explicit, complete, and accurate nonlinear dynamic model of the system is developed using assumed mode method. The Lagrangian approach is used to derive the dynamic model of the structure. The model equations are verified with various bang-bang input torque profiles.**

Keywords- *dynamic model; input torque; Lagrangian*

I. INTRODUCTION

Flexible link manipulators are attractive because they avoid the large inertia forces associated with traditional, large-section, rigid-link manipulators. Flexible robot manipulators require less material, are lighter in weight, consume less power, require smaller actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio. Unlike rigid manipulators, the dynamics of manipulators incorporate the effects of mechanical flexibilities in its links.

Link flexibility is a consequence of the lightweight constructional feature in the manipulator arms that are designed to operate at high speed with low inertia. Thus, flexible manipulators undergo two types of motion, i.e. rigid and flexible motion. Because of the interaction of these motions, the resulting dynamic equations of flexible manipulators are highly complex and, in turn, the control task becomes more challenging compared to that for rigid robots. Each flexible link can be modeled as distributed parameter system where the motion is described by a coupled system of ordinary and partial differential equations (PDE).

PDEs and boundary equations of a two-link flexible manipulator system are obtained by matching the shear force and bending moment at the elbow joint, allowing the eigenvalues to be computed without recourse to dynamic formulations [1]. On the other hand, the vibration modes of a generic two-link flexible manipulator are studied as a function of the link, rotor and tip mass distribution. Necessary and sufficient conditions are developed for all vibration modes to exhibit a node at the manipulator.

Various approaches have been developed which can mainly be divided into two categories: the numerical analysis approach

Buffinton and Kane [4] developed equations of motion for flexible robots containing translational motion of elastic members. The specific system investigated is a two-degree of freedom manipulator whose configuration is similar to the well known Stanford arm, whose translational member is regarded as an elastic beam. The AMM and an alternative form of Kane's method are used in the formulation of equation of motion.

Subudhi and Morries [5] have used a combined Euler-Lagrange formulation and AMM approach to model the planar motion of a manipulator consisting of two flexible links and joints. The conventional Lagrangian modeling of flexible link robots does not fully incorporate the bending mechanism of flexible link as it allows free link elongation in addition to link deflection. De Luca and Siciliano [6] have utilised the AMM to derive a dynamic model of multilink flexible robot arms limiting to the case of planar manipulators with no torsional effects. The equations of motion which can be arranged in a computationally efficient closed form that is also linear with respect to a suitable set of constant mechanical parameters have been obtained [7].

This paper presents modeling of a two-link flexible manipulator using Lagrangian technique in conjunction with the AMM. The links are modeled as Euler-Bernoulli beams satisfying proper mass boundary conditions. A payload is added to the tip of the outer link, while hub inertias are included at the actuator joints. Various input torque profiles are also investigated to verify the dynamic model.

II. A TWO-LINK FLEXIBLE MANIPULATOR

In this work, a two-link flexible manipulator moves in the horizontal plane is considered. Fig. 1 shows a two-link flexible robot manipulator system consider in this study. The ith link has length l_i and uniform mass density per unit length ρ_i. The first link is clamped on the rotor of the first motor. The second motor, is attached to the tip of the first link. E and I represent Young modulus and area moment of inertia of both link. A payload is attached at the end-point of link-2. X_0Y_0 is the inertial coordinate frame. θ_1 and θ_2 are the angular position and $v_i(x_i, t)$ are the transverse component of the